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A VARIABLE ATMOSPHERIC-DENSITY MODEL
FROM SATELLITE ACCELERATIONS

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A VARIABLE ATMOSPHERIC-DENSITY MODEL
FROM SATELLITE ACCELERATIONS

by

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(Manuscript received March 30, 1960)

Summary. - Satellite accelerations are analyzed by an empirical formula that relates the product $\rho H^{\frac{1}{2}}$ (ρ = atmospheric density, H = scale height) to the geometric height z, the 20-cm solar flux F_{20}, and the angular distance ψ' from the center of the diurnal bulge. Once the numerical parameters of this formula have been established, tables of ρ and H are computed and a separate formula is derived to represent ρ in function of the same variables.

1. The 20-cm Solar Flux

A survey of the density fluctuations in the upper atmosphere induced by variable solar radiation and revealed in the variable accelerations of artificial satellites was published in Special Report No. 29 (Jacchia, 1959). Two important papers, based partly on the observational material contained in this report, have been prepared for publication since then (Priester and Martin, 1960; Nicolet, 1960). In the first of these, Priester and Martin have subjected the satellite accelerations to a quantitative analysis, using the 20-cm solar flux as one of the parameters, and have deduced atmospheric density profiles for the central regions of the dark and the bright hemispheres for different values of the solar flux. In the second paper Nicolet proposes an isothermal model for the upper atmosphere, in which the varying gradient of the scale height above 250 km is explained by the decrease of the molecular weight of the atmospheric components which undergo a diffusion process; the diurnal effect is attributed to heat conduction.

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The 20-cm solar flux used by Priester and Martin is measured daily at the Heinrich-Hertz-Institut für Schwingungsforschung in Berlin-Adlershof, and shows large mean-level oscillations that are not present, at least to such a large degree, in the 10.7-cm flux used by this writer in his previous analysis. Nicolet has pointed out that this behavior of the 20-cm flux is peculiar, inasmuch as it has no counterpart in the 3.2-cm, 8-cm, 10-cm, 15-cm, 21-cm, and 30-cm fluxes measured at Berlin, Nagoya, Ottawa and Sydney. For this reason he believes that the fluctuations must be spurious, of instrumental origin, and he rejects the correlation found by Priester and Martin. It must be admitted that it is not easy to explain the peculiar behavior of the 20-cm flux; we have to face the fact, however, that the mean-level fluctuations of the 20-cm flux are generally reflected in the satellite accelerations. If so far this writer has refrained from publishing a more definitive analysis of the correlation between solar radiation and upper-atmosphere densities, he hesitated chiefly because of the presence of oscillations in the satellite accelerations that could not be explained on the basis of the 10.7-cm flux - the "erratic fluctuations of unexplained origin," as they are labelled under d) in the list of solar effects on page 4 of Special Report No. 29 (Jacchia, 1959). A look at Figure 1 will show at once that these fluctuations are entirely accounted for when the 20-cm flux is used.

However, we do not want categorically to exclude the possibility that some drift effect might be present in the 20-cm data. We are quite satisfied, however, that the 20-cm flux seems, on the whole, to be more closely related than the 10.7-cm flux to the ultra-short wave radiation responsible for the atmospheric fluctuations.

2. Satellite Accelerations

To derive actual atmospheric densities, accelerations of the following satellites were used:

Satellite	Acceleration Derived By
1958 β 2	Jacchia (1959); Briggs (1959)
1959 α 1	Jacchia and Nigam (unpublished)
1958 α	Jacchia (1959)
1958 δ 2	Y. Kozai (unpublished)

In addition, the accelerations of 1957 β 1 (Jacchia, 1958) and 1958 δ1 (Jacchia, 1959) were analyzed for the oscillations only; no absolute densities were computed in view of the uncertainty in the physical characteristics of these satellites.

The accelerations of 1959 α1 were derived by numerical differentiation of the mean motion of the satellite, which was computed at 2-day intervals by R. Nigam as part of a regular orbital program. Since observations extending over a whole week were used for each orbit and the secular acceleration was assumed to be constant in this interval, these accelerations are necessarily somewhat smoothed out and sometimes outright uncertain--just as those of 1958 α given in Special Report No. 29. The same can be said of the accelerations of 1958 δ2, which were computed in similar manner by Y. Kozai.

In this connection it should be pointed out that the only really homogeneous, accurately determined series of satellite accelerations in existence today is that of 1958 β2. These accelerations were determined by feeding the orbital elements as known functions of time into an orbital-analysis program and by plotting the residuals of each observation. This method was used also for 1957 β1 and 1958 δ1, but the long periods of invisibility, the inhomogeneity of the observational material, and the sporadic distribution and frequent unreliability of the orbital elements made the derivation of accelerations more uncertain for these two objects.

The frequent requests that have been received to supply satellite accelerations on a service basis indicate that it is not generally realized how delicate and elaborate a procedure the computation of accurate accelerations really is. It is the firm opinion of this writer that reliable accelerations cannot be obtained as mere byproducts of orbital computations and that nothing short of an ad-hoc analysis by an expert numerical analyst will yield accurate results.

3. Densities and Scale Heights from Accelerations

Approximate formulae relating satellite drag to orbital elements and atmospheric parameters have been derived by various authors (Sterne, 1958; Groves, 1958; King-Hele, Cook, and Walker, 1959), and give essentially the same results within the limits of observational accuracy. The formula used in this paper was that of King-Hele, Cook, and Walker, namely:

$$\rho_p H_p^{\frac{1}{2}} = - \frac{\sqrt{\frac{2}{\pi}}}{3C_D} \frac{dP}{dt} \frac{m}{AF} \sqrt{\frac{e}{a}} \left[1-2e + \frac{5e^2}{2} - \frac{H}{8ae} \left(1-10e + \frac{7H}{16ae}\right) \right]. \quad (1)$$

The symbols used are defined as follows:

ρ_p = atmospheric density at satellite perigee height,

H_p = atmospheric scale height around satellite perigee height,

C_D = drag coefficient,

$\frac{dP}{dt}$ = secular acceleration of satellite,

m = mass of satellite,

A = effective cross-section of satellite,

F = factor to account for rotation of atmosphere,

e = orbital eccentricity,

a = semi-major axis of satellite orbit.

Free molecular flow was assumed throughout and C_D was always taken to have the value 2. The height z_p corresponding to ρ_p was taken to be above the international geoid. In view of the paucity of the material on high-inclination satellites, no attempt was made to include latitude and seasonal effects in the analysis.

Rather than try to obtain ρ_p for each satellite with an assumed scale height - which would result in laborious iterations after all the material was assembled - we preferred to do all the analysis on the product $\rho H^{\frac{1}{2}}$, which can be derived directly from the accelerations without assumptions (except for nearly negligible terms). Actual densities were derived only at the end, from the final profiles of $\rho H^{\frac{1}{2}}$ in function of z.

4. An Empirical Formula to Describe Atmospheric Variations

It was felt that the simplest way to deal with the various solar effects was to find an empirical formula that would describe the observed phenomena within the limits of observational accuracy. These phenomena are:

(a). A large diurnal effect at great heights, decreasing to almost zero at the 200-km level. The atmosphere bulges out in the direction of the sun, with a noticeable lag due to the rotation of the earth.

(b). Erratic fluctuations in phase with the 20-cm solar flux. The amplitude of these fluctuations is large in the diurnal bulge and small in the dark hemisphere. At the 200-km level the amplitude of the fluctuations is small but nearly independent of the position with respect to the sun.

Effect (a) must be a function of the height z, of the solar flux F and of the angular distance ψ' from the point where the bulge is highest. It was assumed that this point is at the same latitude as the sub-solar point, but lags in longitude by a constant λ . For a point in the atmosphere, whose astronomical equatorial coordinates are α_p, δ_p, we have

$$\cos \psi' = \sin \delta_p \sin \delta_\odot + \cos \delta_p \cos \delta_\odot \cos (\alpha_p - \alpha_\odot - \lambda) \qquad (2)$$

($\alpha_\odot$, $\delta_\odot$ are the equatorial coordinates of the sun).

From dynamical considerations we must infer that the lag λ is not necessarily the same for all heights.

An equation that will satisfactorily describe both conditions (a) and (b) can be written in the form,

$$\rho H^{\frac{1}{2}} = f_o (z) \left[c_1 F_{20}^m + f_1 (z) \; f(\psi') F_{20} \right] , \qquad (3)$$

where $f_o(z)$, $f_1(z)$ and $f(\psi')$ are suitable functions of z and ψ' respectively, and c_1 and m are constants. F_{20} is the 20-cm solar flux. We shall make $f(\psi')$ vary from 1 to zero where ψ' increases from 0° to 180°.

At 180° from the diurnal bulge ($\psi' = 180°$) the second term inside the brackets will then be zero and we can define $f_o(z)$ as a standard night profile of $\rho H^{\frac{1}{2}}$ in function of z for a fixed value of F_{20}. The function $f_1(z)$ is the amplitude of the diurnal effect and must be small for z close to 200 km; from satellites with perigee at this height we can thus determine the value of the exponent, m, which was introduced to fit the observed amplitude of the erratic fluctuations at low heights.

For satellite 1957 β1 a value of m = 0.7 seems to be satisfactory, while for 1958 δ1 and 1958 δ2 we find that m = 1 gives a sufficiently good fit. For simplicity we have assumed, provisionally, m = 1.

A function which can describe a diurnal bulge of any degree of sharpness is

$$f(\psi') = \cos^n \frac{\psi'}{2} . \qquad (4)$$

The exponent n is best determined from high satellites, where the diurnal effect is large. From satellite 1958 β2, 1959 α1, and 1958 α we find that n = 6 gives a very satisfactory fit.

If we express F_{20} in units of 100 x 10^{-22} watts per square meter per cycle, and define $f_o(z)$ as the night profile of $\rho H^{\frac{1}{2}}$ for $F_{20} = 1$, we can write

$$\rho H^{\frac{1}{2}} = f_o(z) F_{20} \left[1 + f_1(z) \cos^6 \frac{\psi'}{2}\right] . \qquad (5)$$

Table 1 gives the values of $f_o(z)$ and $f_1(z)$ which were deduced from the various satellites, together with the best value of the lag angle λ. All quantities are expressed in the c.g.s. system, except the perigee heights z_p, which are in kilometers.

Table 1

BASIC ATMOSPHERIC DATA FROM SATELLITES

Satellite	z_p (km)	Observed log $f_o(z)$	Observed $f_1(z)$	λ	Computed log $f_o(z)$	Computed $f_1(z)$
1958 β2	655	-12.70	9.0	25°	-12.688	9.05
1959 α1	562	-12.22	4.9	30°	-12.220	5.02
1958 α	353	-10.83	1.2	30°	-10.825	1.17
1958 δ2	210	- 9.46	0.2-0.4	--	- 9.463	0.28

From the observed values, the following expressions were found for $f_o(z)$ and $f_1(z)$:

$$\log f_o(z) = -12.475 - 0.0019 z + 6.01 \exp(-0.0027 z) , \qquad (6)$$

$$f_1(z) = 0.185 \exp(0.006 z - 2) ; \qquad (7)$$

(z is always expressed in km; $200 < z < 700$).

The values of $f_o(z)$ and $f_1(z)$ computed with equations (6) and (7) are to be found, for comparison, at the right-hand side of Table 1.

The final form of equation (4) becomes, then:

$$\rho H^{\frac{1}{2}} = f_o(z) F_{20} \left[1 + 0.185 \exp(0.006 z - 2) \cos^6 \frac{\psi'}{2}\right] . \qquad (8)$$

The lag angle λ in ψ' can be assumed to be between 25° and 30° for all heights; $f_o(z)$ is to be taken from equation (6).

Table 2 gives night and day profiles of ρ and H for $F_{20} = 1$ and $F_{20} = 3$, as computed by equation (8). The ρ profile for $\psi' = 180°$ and $F_{20} = 1$ is well represented by the equation,

$$\log \rho_o(z) = -16.021 - 0.001985\, z + 6.363 \exp(-0.0026\, z) \quad (9)$$

(z is expressed in km; $200 < z < 700$). A general equation for ρ can be written in the same form as equation (8):

$$\rho = \rho_o(z)\, F_{20} \left[1 + 0.19 \exp(0.0055\, z - 1.9) \cos^6 \frac{\psi'}{2}\right]. \quad (10)$$

The numerical coefficients in equations (6), (8), (9), and (10) must, of course, be considered as only provisional and susceptible to considerable improvement when more satellite acceleration material is analyzed.

Table 3 gives height profiles of the diurnal bulge for $F_{20} = 2$; i.e., the height at which a given density is reached as a function of the angular distance ψ' from the center of the bulge. These profiles are illustrated graphically in Figure 3. Figures 1 and 2 show a comparison between observed and computed accelerations for satellites 1958 β2 and 1959 α1.

Remarks and Conclusions

The approach used in this paper is primarily descriptive, and it was felt that theoretical inferences on atmospheric parameters other than the density should be left out, since they cannot be derived without some degree of speculation. Equation (10), or improved versions of it, should prove useful for a quick computation of corrections to be applied to a standard atmospheric density profile at a given time and place.

Due to the nature of the problem, the relative accuracy of the densities derived from our formulae is considerably greater than that of the corresponding scale heights - in particular, at the two extremes of the height zone covered by satellites. A critical point is the one around 200 km, where the density profile derived from satellites should be connected with that obtained from rockets. This operation may not be very difficult for a mean density profile; when, however, a particular profile such as our ρ_o (z) is chosen, it becomes necessary to know how the amplitude of the erratic fluctuations related to F_{20} varies with height in the region from 100 to 200 km.

It should not be forgotten, of course, that the diurnal effect - or any other atmospheric effect dependent on geographical position - is bound to be somewhat smoothed out or even distorted in its action on satellite motions, especially if the orbital eccentricity is small (fortunately this is not true of the higher satellites investigated in this paper). This situation occurs because the formulae used to derive the atmospheric density at perigee assume that the atmosphere is spherically symmetric.

Equation (10) consists of two terms, of which the first, $\rho_0(z)\ F_{20}$, is independent of geographical position and does not affect the scale height of the atmosphere. This would imply an absorption of the effective solar radiation that takes place entirely below the 200-km level and a uniform distribution of the absorbed energy throughout both the bright and the dark hemispheres. The second term of equation (10) vanishes at $z = 117$ km, when $\exp(0.0055\ z) = 1.9$. The location of this vanishing point is highly uncertain, but it is tempting to identify this height with the mean height of the absorption zone. The diurnal effect can easily be explained by the mechanism proposed by Nicolet (1960).

References

BRIGGS, R. E.

1959. A table of times of perigee passage for Satellite 1958 Beta Two. Special Report No. 30, Smithsonian Astrophysical Observatory, November 12, 1959.

GROVES, G. V.

1958. Effect of the earth's equatorial bulge on the lifetime of artificial satellites and its use in determining atmosphere scale heights. Nature, vol. 181, p. 1055.

JACCHIA, L. G.

1958. Orbital results for Satellite 1957 Beta One. Special Report No. 13, Smithsonian Astrophysical Observatory, May 21, 1958.

JACCHIA, L. G.

1959. Solar effects on the acceleration of artificial satellites. Special Report No. 29, Smithsonian Astrophysical Observatory, September 21, 1959.

KING-HELE, D. G., COOK, G. E., AND WALKER, D. M. C.

1959. Contraction of satellite orbits under the influence of air drag, Part 1. Royal Aircraft Establishment (Farnborough), Technical Note No. G. W. 533.

NICOLET, M.

1960. Les variations de la densité et du transport de chaleur par conduction dans l'atmosphère superieure. Centre National de Recherches de l'Espace, Bruxelles. Note Preliminaire No. 5.

PRIESTER, W. AND MARTIN, H. A.

1960. Solare und tageszeitliche Effekte in der Hochatmosphäre und Beobachtungen an Künstlichen Satelliten. Mitteilungen der Universitäts-Sternwarte Bonn, Nr. 29 (in press).

References

STERNE, T. E.

1958. Formula for inferring atmospheric density from the motion of artificial earth satellites. *Science*, vol. 127, p. 1245.

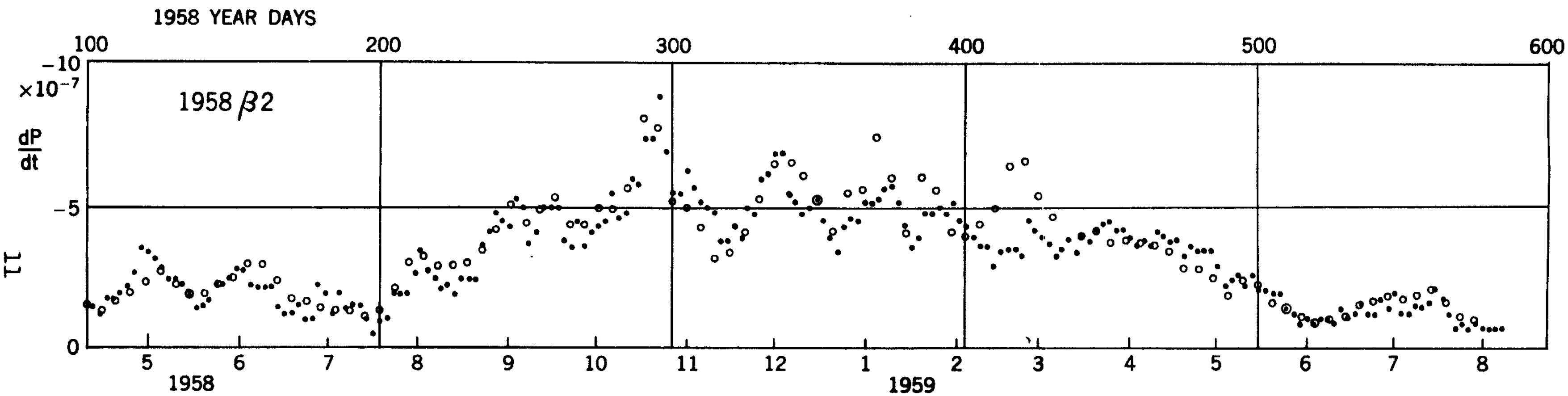

Figure 1. Observed accelerations of Satellite 1958 $\beta 2$ (dots) compared with accelerations computed by equation (8) (open circles), with lag angle $\lambda = 25^{\circ}$.

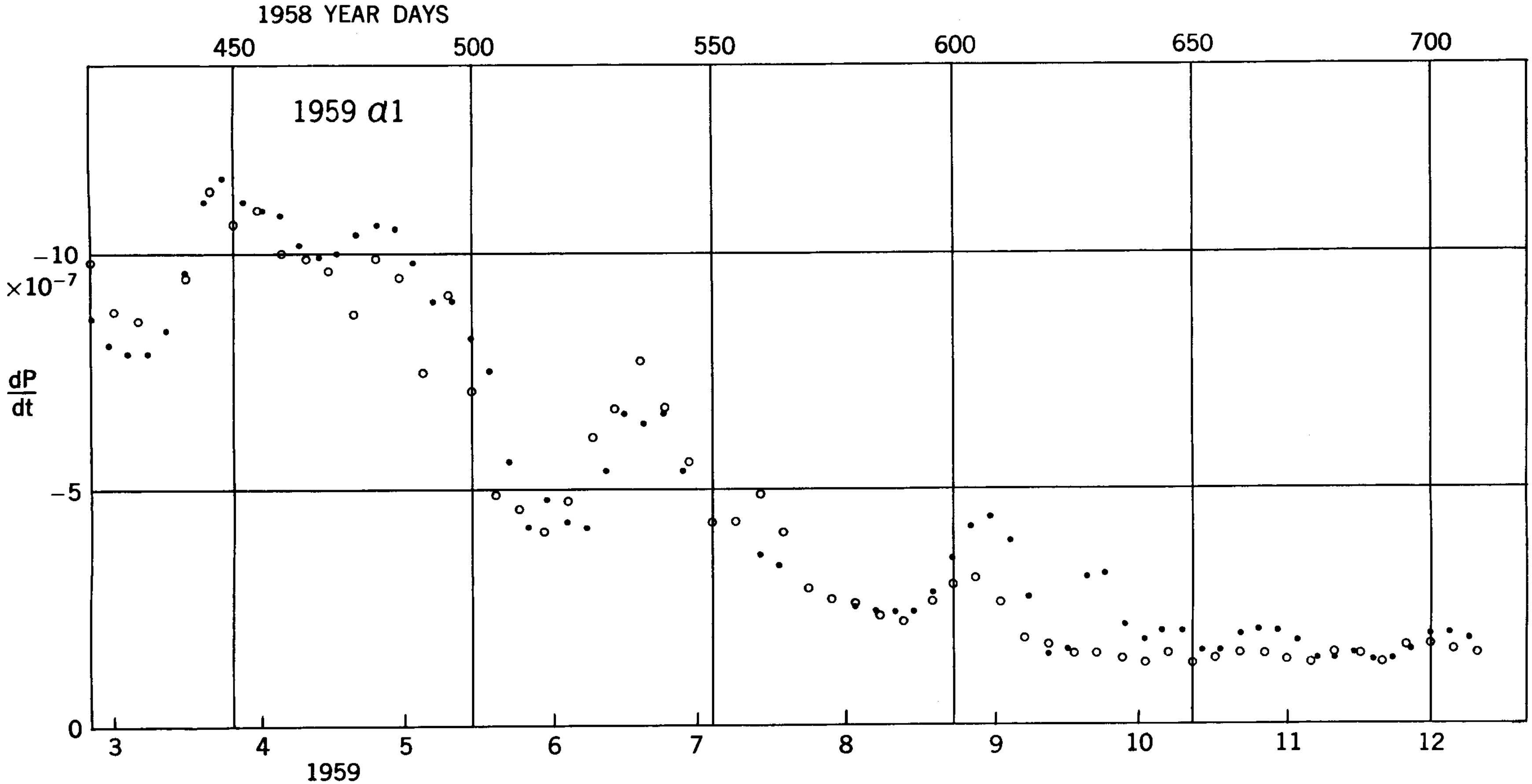

Figure 2. Observed accelerations of Satellite 1959 α 1 (dots) compared with accelerations computed by equation (8) (open circles), with lag angle $\lambda = 30^{\circ}$. No 20-cm flux data were available after Aug. 31, 1959 (= 1958 Year Day 608); after that date the 10.7-cm flux from Ottawa was used, multiplied by an empirical factor 0.85 to adjust its mean level to that of the 20-cm flux.

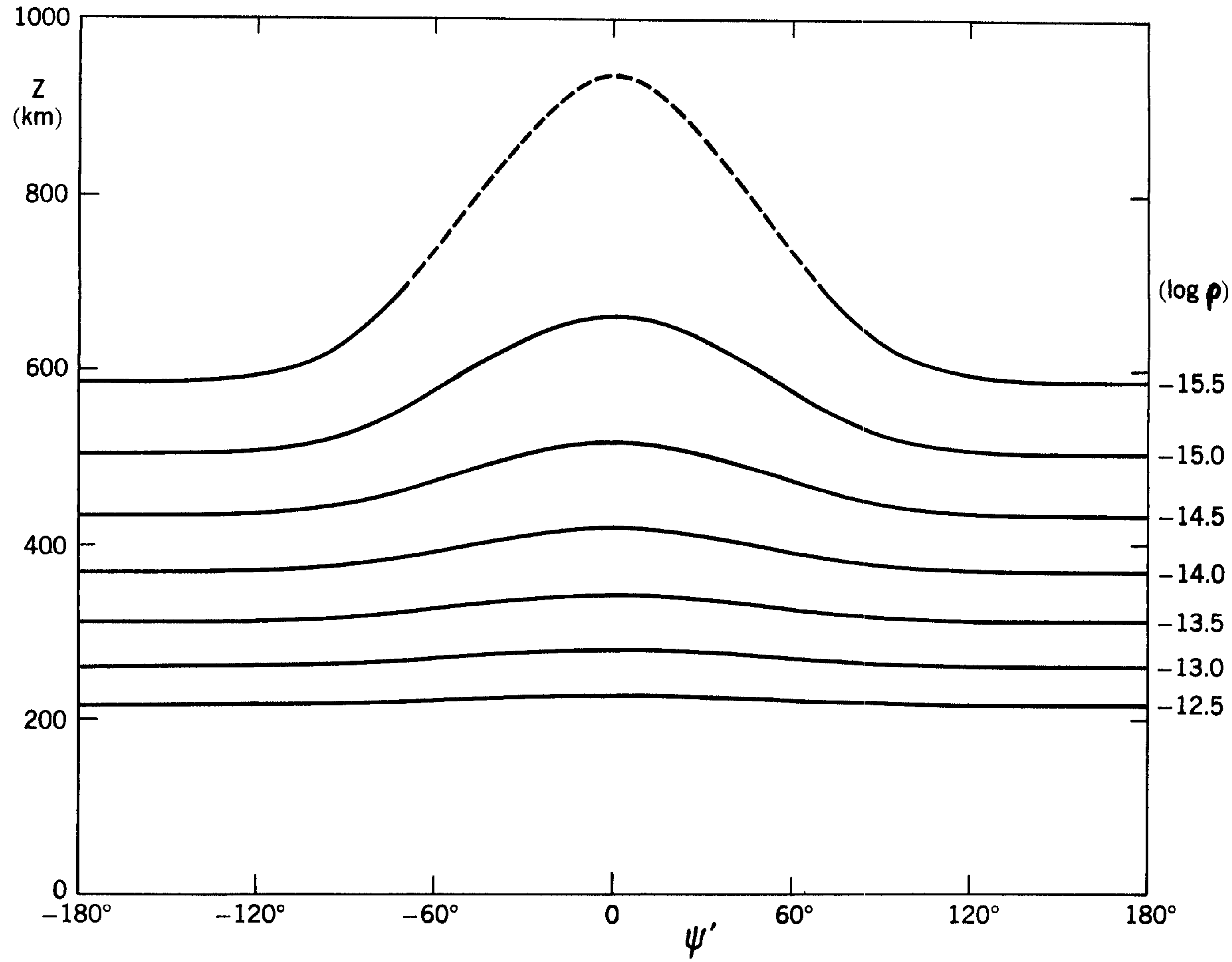

Figure 3. Height of surfaces of equal density above a great circle across the diurnal bulge, computed for a 20-cm solar flux of 200 x 10^{-22} watts/m^2 cycle. The value of log ρ corresponding to each curve is marked on the right-hand margin.

TABLE 2

ATMOSPHERIC DENSITY PROFILE COMPUTED FROM EQUATIONS (6) and (8)

	Night ($\psi' = 180^\circ$)				Day ($\psi' = 0^\circ$)			
z (km)	$\rho H^{1/2}$ $F_{20} = 1$ [$= f_o(z)$]	H (km)	$\log \rho$ $F_{20} = 1$	$\log \rho$ $F_{20} = 3$	$\rho H^{1/2}$ $F_{20} = 1$	H (km)	$\log \rho$ $F_{20} = 1$	$\log \rho$ $F_{20} = 3$
200	-9.353	36.6	-12.635	-12.158	-9.258	40.7	-12.563	-12.086
220	-9.575	38.3	-12.867	-12.389	-9.453	43.0	-12.770	-12.293
240	-9.787	40.0	-13.088	-12.611	-9.638	45.5	-12.966	-12.489
260	-9.990	41.8	-13.301	-12.824	-9.811	48.1	-13.152	-12.675
280	-10.185	43.6	-13.505	-13.028	-9.975	50.9	-13.328	-12.851
300	-10.371	45.4	-13.700	-13.223	-10.128	53.8	-13.494	-13.017
320	-10.550	47.3	-13.887	-13.410	-10.273	57.0	-13.651	-13.174
340	-10.721	49.3	-14.067	-13.590	-10.409	60.4	-13.799	-13.322
360	-10.885	51.3	-14.240	-13.763	-10.536	63.9	-13.939	-13.462
380	-11.043	53.4	-14.406	-13.929	-10.656	67.7	-14.071	-13.594
400	-11.194	55.5	-14.566	-14.089	-10.768	71.8	-14.195	-13.718
420	-11.339	57.7	-14.720	-14.243	-10.872	76.1	-14.313	-13.836
440	-11.479	59.9	-14.868	-14.391	-10.971	80.7	-14.424	-13.947
460	-11.613	62.2	-15.010	-14.533	-11.063	85.5	-14.528	-14.051
480	-11.742	64.5	-15.147	-14.670	-11.149	90.6	-14.627	-14.150
500	-11.867	66.9	-15.280	-14.802	-11.229	96.1	-14.720	-14.243
520	-11.987	69.3	-15.407	-14.930	-11.304	102.0	-14.808	-14.331
540	-12.102	71.8	-15.530	-15.053	-11.374	108.2	-14.891	-14.414
560	-12.214	74.3	-15.649	-15.172	-11.439	114.8	-14.968	-14.491
580	-12.322	76.8	-15.764	-15.287	-11.500	121.7	-15.042	-14.565
600	-12.426	79.4	-15.875	-15.398	-11.556	129.0	-15.111	-14.634
620	-12.526	82.0	-15.983	-15.506	-11.609	136.8	-15.177	-14.700
640	-12.623	84.7	-16.087	-15.610	-11.658	145.0	-15.239	-14.762
660	-12.718	87.4	-16.188	-15.711	-11.703	153.7	-15.297	-14.820
680	-12.809	90.0	-16.286	-15.809	-11.745	162.9	-15.352	-14.875
700	-12.897	92.7	-16.381	-15.904	-11.784	172.7	-15.404	-14.927

TABLE 3

HEIGHT PROFILE (KM) OF SURFACES OF EQUAL DENSITY IN

FUNCTION OF THE ANGULAR DISTANCE ψ' FROM THE CENTER OF THE DIURNAL BULGE

$\log\rho$ \ ψ'	0°	15°	30°	45°	60°	75°	90°	120°	150°	180°
-15.5	938.9	919.6	868.9	802.1	733.3	673.5	630.1	592.0	586.7	586.6
-15.0	662.2	655.2	635.7	607.4	575.9	547.1	525.6	506.1	503.4	503.3
-14.5	518.0	514.2	503.5	487.9	470.6	454.8	443.1	432.4	430.9	430.9
-14.0	418.0	-	409.3	-	389.9	-	374.1	368.2	367.4	367.2
-13.5	340.8	-	335.5	-	323.9	-	314.7	311.2	310.8	310.7
-13.0	277.7	-	274.5	-	267.6	-	262.3	260.3	260.0	260.0
-12.5	224.0	-	222.2	-	218.4	-	215.5	214.4	214.3	214.3

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ERRATA

In Special Report No. 39 of the Smithsonian Astrophysical Observatory the right-hand parenthesis appears misplaced in equations (7), (8), and (10). The equations should read:

$$f_1(z) = 0.185 \left[\exp (0.006\ z) - 2\right] \qquad (7)$$

$$\rho H^{1/2} = f_o(z)\ F_{20} \left\{1 + 0.185 \left[\exp\ (0.006z) - 2\right] \cos^6 \frac{\phi'}{2}\right\} \qquad (8)$$

$$\rho = \rho_o(z)\ F_{20} \left\{1 + 0.19 \left[\exp\ (0.0055z) - 1.9\right] \cos^6 \frac{\phi'}{2}\right\} \qquad (10)$$

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